Multi-Cell Causal Discovery For Entity Specific Latent Variables In Edge Localized Smart Sensors

Chaitanya Kapoor [1] , Sethuraman Rao [2]
kapoorchaitanya42@gmail.com, sethuramanrao@am.amrita.edu
1 - Department of Computer Science and Engineering,
2 - Center for Wireless Networks and Applications
Amrita Vishwa Vidyapeetham
Amritapuri, India

***Abstract*—Many approaches to tackle causal discovery have been discovered until now. The main focus through such a framework is to establish a relation between the cause and the effect. In our work we have implemented a model to explain how causal discovery can be established between different sensor nodes in a smart city setup to automatically establish communication amongst peripheral nodes in a defined sub-network. By first establishing the communication protocol for a sub-network in a smart city, we have scaled it up to multiple sub-networks present in the city topology. From those nodes, we scale the communication to all the defined sub-networks in the city. Establishing causal relations for sub-networks representing individual cells has been the focus of our work.These causal relations have been used to implement a node retrieval protocol for a smart city sensor discussed in our previous work [1].**

***Index Terms*—Causal Discovery, Smart Cities, Multi-Cell System, Sub-networks, Internet of Things (IoT)**

I. Introduction

Causal Discovery is a technique that can be used to build a framework for establishing causal relationships[16]. These relationships are formed through observational data, and there are a variety of reasons that might lead to the formation of these relationships. A causal model is programmed to predict how a system will act by predicting its dynamics. The truth value in this stipulation relies solely on the the prediction of the intervention effects. While these are counterfactual assertions relating to the system, it's safe to say that the probabilistic nature of variables represented in such a model are all aspects of a causal model. Since probabilistic correlations can be established among variables as the results of experimental interventions, causal models can also be inversely used to predict whether they are consistent with them.

Smart cities comprise of multi-modal application of IoT systems and 5G networks to produce a low latency framework that can enable quick and resilient communication. 5G resource allocation frameworks are often closely linked with dynamic edge computations that support such communication frameworks. In our previous work [1], we have discussed such a smart city framework that uses a MIMO based communication framework to enable interactions between various infrastructure units and other units in the city framework. In our current work we focus on establishing a causal discovery based mechanism for identifying sensor nodes at different hierarchies of the smart city framework .The mechanism is further extended to an advanced groundwork for identifying latent variables relevant to the smart city topology.

In our previous work [1] we have explained a hexagonal city framework communicating using MIMO communication protocol. The city framework operates based on a hierarchical distribution of sensor nodes operating on the cloud. The sensor node distribution in the city comprises of three types of nodes, edge cloud node, fog cloud node and the central cloud node. These are distributed in a way to facilitate efficient communication protocols between different entities of the city topology. In the current work, we have built upon the previous work to add a mechanism for causal discovery to recognise the node mapping of all sensor nodes in the topology. This has been further applied to understand nodes that are non operational in the city owing to a disaster.The left figure in Figure 1 shows a detailed account of the proposed topology in the first paper, while the links shown to the sensor distribution in the right figure show the cluster systems proposed in this paper.

The paper has been structured in the following way. The related work section discusses the papers that were relevant to the causal discovery model proposed in this paper. The next section discusses in detail about the causal discovery model, discussing about the topology details, the methodology to establish causal cues, the experiment details to estimate the latent variables and finally the mathematical formulation for identifying the node mapping for the smart city topology. The next section discusses relevant details of the implementation for the experiment and the results obtained through the proposed causal discovery model and is followed by the conclusion section finally.

II. Related Work

[4] presents a framework for group identification based on causal discovery. They propose a specific and shared causal model (SSCM) that accounts for the variability of causal links among individuals/groups while leveraging their

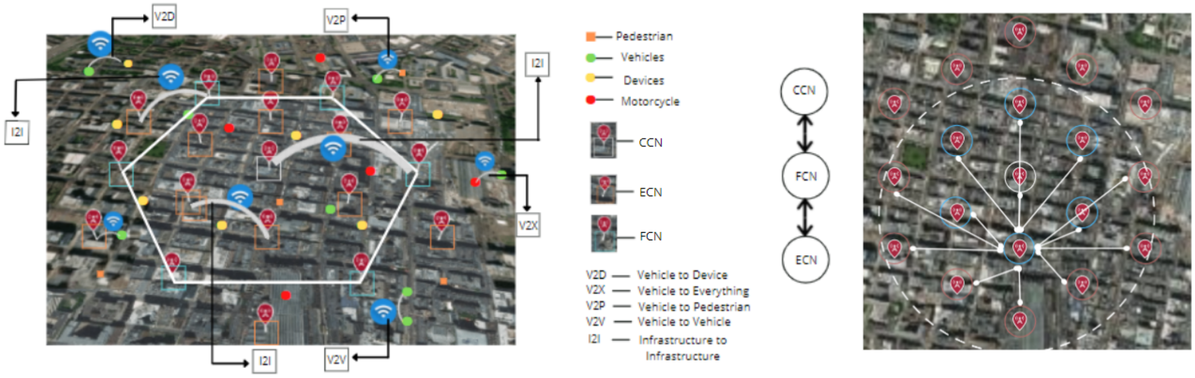

Figure 1. The city topology as implemented in [1]. The extended network topology on the right shows the FCN sensors[blue] and the ECN sensors[yellow]

commonalities to achieve statistically valid estimation. In most state-of-the-art approaches to causal discovery, an underlying causal model is assumed to be fixed. However, causal models frequently differ between domains or people as a result of potentially missing elements that impact the quantitative causal effects. The learnt SSCM provides particular causal knowledge for each individual as well as a population-wide trend. Furthermore, the calculated model directly offers each individual's group information. The proposed method's efficacy is demonstrated by experimental findings on synthetic and real-world data.

To use Structural Causal Models (SCMs) for counterfactual reasoning, understanding the causal mechanisms that yield factorization of the corresponding conditional distributions deterministic functions that map the noise realisations to individual samples. [3] propose that when evaluating counterfactual treatment effects, the optimum causal mechanism should be chosen based on quantitative criteria such as variance minimization. A parameterized family of

Gumbel-max-like causal processes is shown. They show that they can be trained to minimize counterfactual effect variance and similar losses on a distribution of interest inquiries, resulting in lower variance estimates of counterfactual treatment impact than fixed alternatives and generalising to queries not encountered during training.

III. Causal Discovery Model

In our previous work[1], we have proposed a hexagonal based city topology comprising of three hierarchical units. These were the fog cloud node [FCN], the edge cloud node [ECN] and the central cloud node [CCN]. The overall topology proposed in the previous work focuses on establishing I2I (Infrastructure to infrastructure), and V2X (vehicle to everything) communications. The paper also discusses mechanisms for node retrieval in case a node becomes dysfunctional in the case of a disaster. In this work, we have focused on establishing causal relationships between inter-sensor data exchanges. In the modelling of the topology, sending of a message can be identified as a cause and the corresponding effect would the receiving of the message. The nodes that have been identified with the data exchange hence form the base for establishing the causal links for the smart city topology. In our work, the exchange of sensor data essentially comprise the basis of the causal relationship amongst the sensors in the topology.

A. Topology Details

In the proposed smart city topology, one sub-network has been defined as a part of the city topology comprising of a CCN sensor in the center of the topology, while the FCN sensors to be distributed on the corresponding hexagonal edges on the vertices of the hexagon. These sensors are ideally considered to be presented on road intersections. ECN sensors have been defined to be distributed on the side of corresponding roads, hence enabling V2I (vehicular to infrastructure communication) protocols. I2I communication as proposed in [1] can be carried out between any sensor unit.

A cluster is defined as consisting of the ECN, and CCN sensors in the periphery of an FCN node. This is essentially established for the nodes that will be in the range of the FCN sensor (refer to figure 1). In the topology, we have identified the causal relationship in the form of cause and effect for the sender and the corresponding receiver node. This way, the modelling has been carried out by assuming all the sensor nodes are represented by the notation $\mathbb{T}=\left\{\left(\boldsymbol{x}^{(n)}, \boldsymbol{y}^{(n)}\right)\right\}_{n=1}^{N}$. Here x is used to represent the sender sensor and y represents the receiver sensor. In this notation, the periphery sub-network sensor nodes have also been considered.

All the entities of the clusters hence respond to the parameter γ , that represent the state (active or inactive) of the sensor. This way, the total number of active sensors in the sub-network can be allocated in a group of sensors represented by C $\in$ $\{1,\ldots,$ c$\}^N$ that A parameter E represents the total number of entities defined for c clusters in the sub-network topology. This way, clusters have been created in the sub-network group based on their state. Based on the parameter γ, all the entities belonging to one of the defined C clusters have been defined over $\mathcal{N}\left(\mu_c, \sigma^2\right)$.

Defining the likelihood over a function E, the likelihood clustering has been carried out for the function $\mathcal{C}_{\mathcal{E}}$, to represent

the likelihood of clusters defined over the entities in the smart city.Hence the likelihood estimate for the entities in the cluster can be understood from the following equations :
$\mathcal{C} = \{\mu_c\}_{c=1}^{C}$, and $\mathcal{C}_{\mathcal{E}} = \ell(\mathcal{C}, \mathbf{E})$

$\mathcal{C}_{\mathcal{E}} = \log \prod_{E=1}^{c} \left\{ \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{1}{2\sigma^2}\left(\gamma_n - \mu_c\right)^2\right)\right\}$

This idea has been extended in the upcoming sections for extending the node retrieval protocol discussed in the previous work and establishing the causal relations for the sensors. Figure 1 showcases the implementation as carried out in the previous work. The network topology has the added representation of the sub-network implementation. The figure shows the representation of one cluster in the topology.

B. Establishing Causal Cues

In a part of the city topology, for instance junctions as explained in [1], the proposed hexagonal structure would follow. But in a city, there would be several such junctions that would correspond to several more sub-networks other than the 6 sub-networks that would be created based on the topology at one junction. We define causal cues as the correlations in the data exchange for clusters in the various sub-networks. Similarly, there might be various clusters that are common to many sub-networks. Therefore, this establishes the need for an automated strategy to implement causal relations between multiple sub-network clusters. This way, the devised algorithm for causal cues can be used to perceive the communication trends over inter- sub-network clusters.

For the entire topology, we have assumed n number of total clusters identified for a total number of k S_{net} sub-networks, we have focused on modelling the causal cues of one cluster over the entire sub-network. Assuming low latency for the data exchange, we are considering all communication protocols to be operating without any time lag. P_i has been defined as the set of immediate causes based on the cluster interactions in a sub-network. We define the causal cue for a sensor entity S_c belonging to a cluster of the sub-network S_{net_k} in the city topology at a time instant t,to be generated as the process[4] :

$$S_c(t) = \underbrace{\sum_{j\in\mathcal{P}_i} S_{net_k}}_{\text{total causal discovery}} = \underbrace{\sum_{p=1}^{p_l}\sum_{j\in\mathcal{L}_i} S_{ij,p}}$$

$$P\left(S_{cij}\right) = \sum_{k=1}^{q} P\left(z_k = 1\right) P\left(S_{cij} \mid \mu_{k,ij}, \sigma_{k,ij}\right),$$

The second equation explained above strength from x_1 to x_2, e_1 and e_2 denote the noise where $P\left(b_{ij} \mid \mu_{k,ij}, \sigma_{k,ij}\right) = \mathcal{N}\left(b_{ij} \mid \mu_{k,ij}, \sigma_{k,ij}^2\right)$ and $\mathcal{N}(\cdot)$ denotes a terms w.r.t x_1 and x_2, referring to the Gaussian distribution, $P\left(z_k = 1\right) = \pi_k$, and $\sum_{k=1}^{q} \pi_k = 1$.The term S_{cij} represents the instantaneous causal influences from the overall variable j to i for the S_{net} cluster. z_k has been used for the $k_t h$ cluster representation in the distribution. This approach[2] suggested a measure I to quantify linear causal effect (CE) of perturbation to further

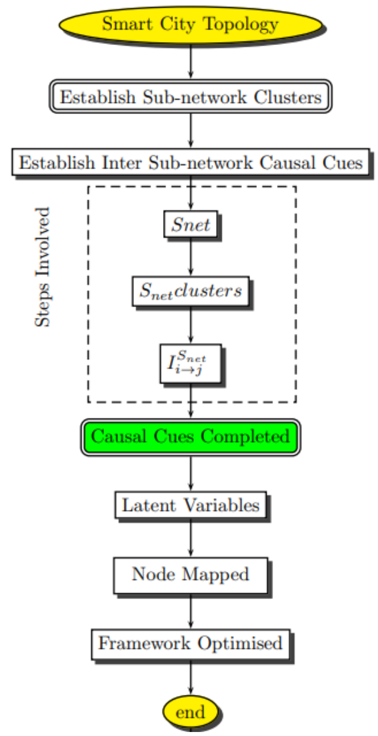

Figure 2. Algorithm for latent variable modelling

quantify causal interaction between the sub processes.

$$I_{i\to j}^{CE}(\tau) = \Psi_{ji}(\tau)$$

where $\Psi'(\tau)$ represents the iterative computed matrix products of the corresponding estimated coefficient matrices $\Phi(\tau)$ by

$$\Psi(\tau) = \sum_{s=1}^{\tau} \Psi(s)\Phi(\tau - s)$$

The sum over the products of path coefficients exclusively along causal paths through k is the mediated causal effect (MCE) through a component k.

$$I_{i\to j}^{MCE}(\tau) = \Psi_{ji}(\tau) - \Psi_{ji}^{(k)}(\tau)$$

By implementing the hence discussed clustering mechanism, we have focused on comparing our implementation with existing one's over real-time and synthetic data-sets. The results of the simulations have been discussed in the sections below.

C. Estimating Latent Variables

In the case of a network topology, one of the crucial factors for effective communication is the resilience corresponding to its nodes. In case of a node failure, effective mechanism to ensure quick mitigation of node damage is very important. We have considered the resilience of the sensor nodes as the

latent variable for the smart city topology. Using a generalized Gumbel-max coupling mechanism[3], we have focused on establishing causal mechanism to extend the node retrieval protocol discussed in the previous work.

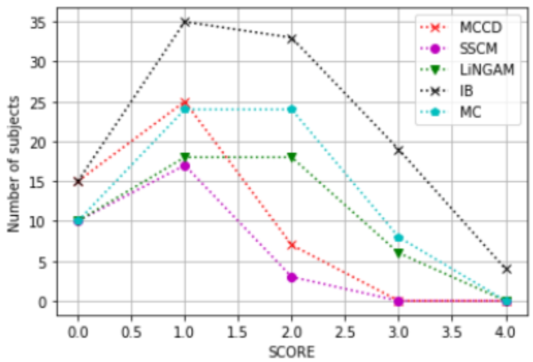

Figure 3. Initial Causal Graph-1

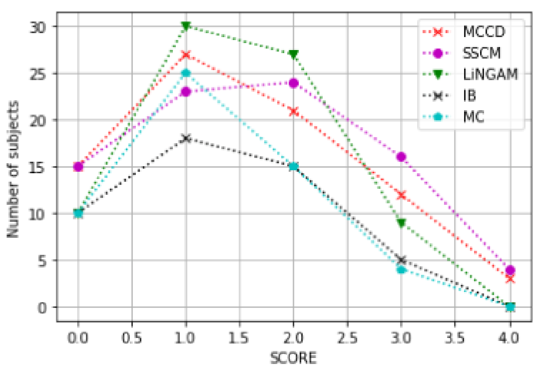

Figure 4. Initial Causal Graph-2

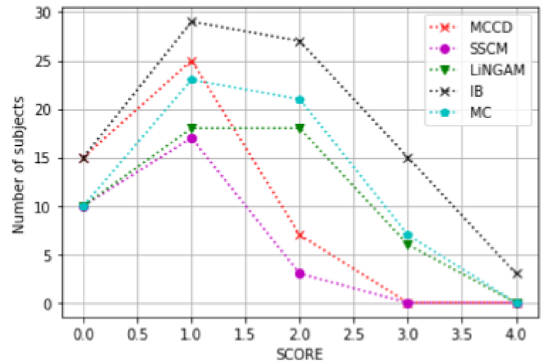

Figure 5. Recovered Causal Graph-1

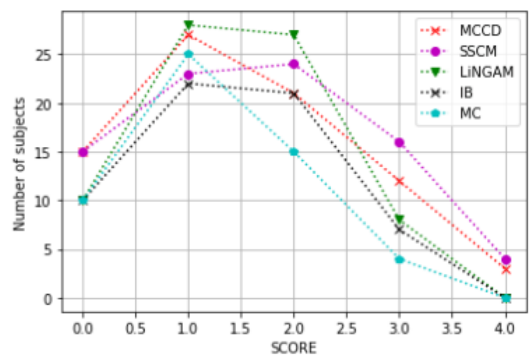

Figure 6. Recovered Causal Graph-2

We have taken forward the theoretical framework extended in [1] and incorporated the coupling technique discussed in [3]. This way, in the case a node is identified as not working, the implemented mechanism lays down a foundation for CCN node to identify the faulty node using a coupling technique. The discussed mechanism comprises of two steps. The first step essentially is to identify the corresponding node mapping for the sensor distribution in the smart city topology.

D. Identifying node mapping For Smart City topology

Based on [5] we have implemented a mapping representing the network distribution of the nodes in the smart city as the following representation:
$\pi_\theta : \mathbb{R}^{S_{net}} \rightarrow \mathbb{R}^{S_{net} \times S_{net}}$. Hence the entity network in all the sub-networks is reduced from a categorical distribution $p \in \mathbb{R}^{S_{net}}$ to an auxiliary joint distribution $\pi_\theta(x, z \mid p)$ for the entire city topology which is represented as a matrix $\pi_\theta(\cdot, \cdot \mid p) \in \mathbb{R}^{S_{net} \times S_{net}}$. We have used the following equation to

implement a coupling for a sensor distribution for the entire sub-network as a function over the variables x, and y defined in the topology details sub-section of the causal discovery model section. :

$$[\gamma_1(p)]_x{}^n = \max_z \{\gamma_{x^n, z^n} + \log \pi_\theta(x^n, z^n \mid p)\}$$

$$[\gamma_2(q)]_y{}^n = \max_z \left\{ \left(\gamma^T\right)^n_{y,z} + \log_\theta(y^n, z^n \mid q) \right\}$$

$$\hat{x} = \operatorname{argmax}^n \{\gamma_1(p)\}^n$$

$$\hat{y}^n = \operatorname{argmax} \{\gamma_2(q)\}^n$$

The mappings construction is confined in such a way to marginalise away the auxiliary variable 'z'. This provides a distribution that is consistent with the provided nodes, represented by the notation i.e., $\sum_z \pi_\theta(x, z \mid p) = p(x)$. We then generate K^2 independent $\gamma_{x,z} \sim \text{Gumbel}(0)$ samples and perform Gumbel-max on the auxiliary joint.The mappings construction is confined in such a way that marginalising away the auxiliary variable 'z' provides a distribution that $\gamma(p) = \max_z \{\gamma_{x,z} + \log \pi_\theta(x, z \mid p)\}$ and then return $\hat{x} = \operatorname{argmax}\{\gamma(p)\}$. Because this involves doing Gumbel-max on a joint distribution with accurate marginals and then marginalising out the auxiliary variable, $hatx$ is distributed according to p. We execute this process individually for p and q, but with shared realisations of the $K2$ Gumbels, to create a coupling.

IV. Implementation Details and Experiments

In this section we have discussed the details regarding the implementation for the causal discovery. The implemented algorithm has been tested using synthetic data for the city topology. The main goal of the experiment is to carry forward the framework proposed in [1] by incorporating causal discovery. The enhanced framework has been implemented to identify non-functional faulty nodes in the city topology. These nodes could be rendered faulty due to a disaster or any other reason. We first explain the experiment setup to carry out the causal discovery in the topology and then discuss the details of the results obtained.

A. Experiment

The efficacy of the implementation for causal discovery discussed in the previous section has been tested using synthetic simulation.The synthetic data generation has been carried out following the model proposed in [4] with a parameter of 0.3. The generated causal structures follow acyclic style and the graph structure is being represented by G. Each graph generated has n variables. These have been carried out for samples size a,b,c of the number of clusters. These group cases have been generated to support a real-time scenario of cluster generation.

For the causal discovery, we have focused on the identification of causal relation based on the proposed approach. The proposed technique in the paper has been referred to as MCCD - Multiple Cell Causal Discovery. For understanding the efficiency of the algorithm, we have compared our results for synthetic data with benchmarks related to similar work in the field. For verifying the results we have focused on establishing the F1 score to measure the accuracy obtained on F1 graph. These accuracy are essentially a score with which the provided methods can replicate a learned graph. In the results discussed in figure 4, we compare the established model with 4 other baselines for F1 scores, where F1 score refers to the ability to learn the graphs efficiently. These baselines are SSCM[4], LiNGAM[14], IB[15], and MC[15]. The results clearly show our model outperforms other benchmarks.

Through the algorithm discussed in the previous section, the sensor distribution in the city topology can be understood. Using the topology details understood from the city mapping, we carried out experiments to analyse the sensor node details.

B. Discussion

In this section the results obtained in the experiment section have been discussed. Figure 3,4,5,and 6 showcase the results of the simulations. The figure 5, and figure 6 showcase the results of the recovered causal graphs represented by Figure 3, and Figure 4. As shown in the figures.

1) Graph-1: In the case of graph-1, it can be observed that the recovered graph in the case of our proposed methodology, MCCD achieves almost the same causal graph in figure -5 as followed in figure 3. The graph recovery performance in the case of MCCD also outperforms the performance of other techniques.

2) Graph-2: In the case of graph-2, it has been observed that the recovered graph in the case of our proposed methodology, MCCD achieves almost the same causal graph in figure-6 as followed in figure-4. The graph recovery performance in the case of MCCD also outperforms the performance of other techniques.

V. Conclusion

Improving on the the theoretical framework established in our previous work, we have modelled a more advanced technique using causal discovery to locate the exact non-functional nodes in the topology. The established model can be used to implement communication in a Multi-Cell city topology using causal cues as discussed in the paper.The discussed idea can be very useful in real-time implementation relating to disaster mitigation.We have used various techniques to establish the discussed formulation. The proposed city topology hence is able to recognise faulty nodes and come up with a mechanism to mitigate the problems.

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